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GeoGebra Lesson: Using Riemann Sum and Integration to Find the Area Under a Curve

1) Either launch the GeoGebra desktop application from your computer or access it in a web browser with the following URL: http://web.geogebra.org/app/

2) Enter the following equation into the input box at the bottom of the screen:

f(x)=a*(x^2)+bx+c

When the program asks if you would like to create sliders for **a**, **b**, and **c**, say yes. This will create a parabola on the graph; scroll over and center the graph.

3) Right-click on the sliders for **a**, **b**, and **c**. Go to object properties and choose the slider tab. Set the increment to **1** for each.

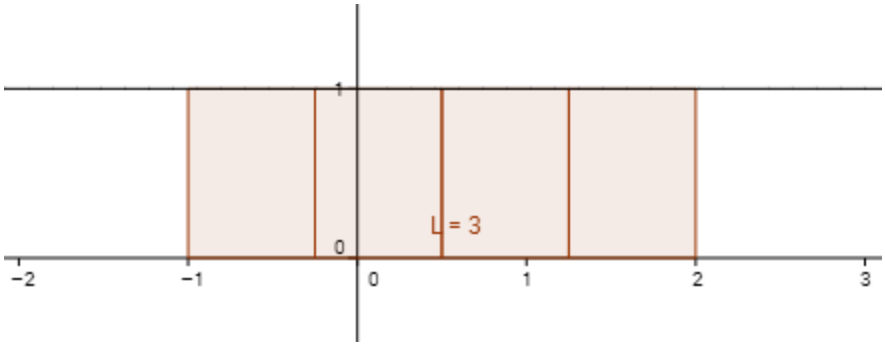
4) Return to the input box and enter the following equations:

U=UpperSum(f,x_1,x_2,n)
L=LowerSum(f,x_1,x_2,n)
F=Integral(f,x_1,x_2)

Allow the program to create sliders for **x_1**, **x_2**, and **n**.

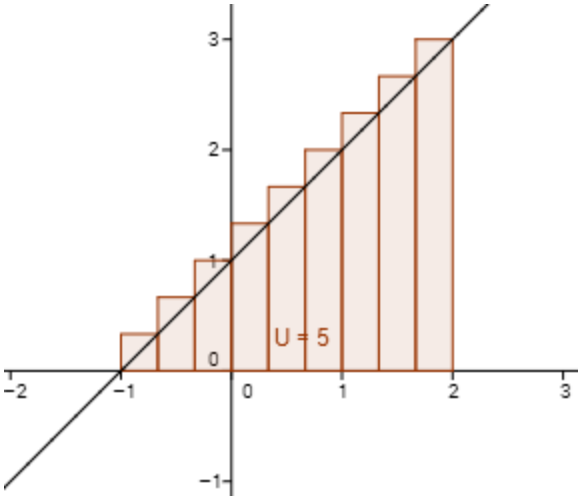
5) Right click on the sliders for **x_1** and **x_2** to change the increment to **1** for each. Right click on the slider for **n** to give it an increment of **1**, minimum of **1**, and maximum of **50**. You should now have sliders that control the shape of the graph, the limits of integration, and the number of rectangles under the curve (for calculating the Riemann sum). Note that you can click on the buttons next to **F**, **L**, and **U** on the left to observe them individually. It may also help to zoom in.

6) Move the sliders for **a** and **b** to **0**. Move the slider for **c** to **1**. Move **x_1** to **-1** and move **x_2** to **2**. You should get a shape like this:



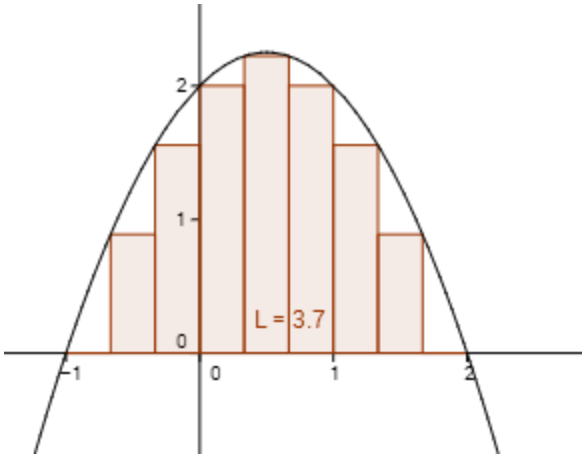
- What do you notice about the values for **U**, **L**, and **F**?
- What happens if you use the slider to change the value of **n**?
- Which formula could you use to find the area of this shape?
- Calculate the area using this formula.

7) Move the slider for **b** to **1**. You should get a shape like this:



- What do you notice about **U**, **L**, and **F** now?
- What happens when you move the slider for **n** to low values? High values?
- Which formula could you use to find the area of this shape?
- Calculate the area using this formula.

8) Move the slider for **a** to **-1** and the slider for **c** to **2**. You should now see this shape:



- What do you notice about **U**, **L**, and **F** now?
- What happens when you move the slider for **n** to low values? High values?
- Could you come up with a formula for the area of this shape?
- Calculate the area using an integral.

When calculating the area under a curve using the Riemann sum method, greater values of **n** will give you a more accurate answer; upper sums will overestimate the area while lower sums will underestimate it. The greater the number of rectangles, the lower the margin of error will be. Intuitively, this makes sense; integration is the equivalent of summing infinitely many rectangles, each of which is infinitely thin. Continue to play around with the sliders and see what other observations you can make... it's educational *and* fun!

This lesson was adapted from the following online guide:
http://www.geogebra.org/workshop/en/GerritStols-GeoGebra-in10Lessons.pdf